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One-Parameter Coherent State Representation of the spl(2, 1) Superalgebra

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Abstract One-parameter general coherent states of the spl(2, 1) superalgebra are constructed and their properties are discussed in detail. One-parameter matrix elements of the spl(2, 1) generators in the one-parameter general coherent-state space are calculated. The parameter α may be related to the interaction parameter U in one exactly solvable model for correlated electrons.

Keywords spl(2, 1) superalgebra · Coherent state · Exactly solvable model

1 Introduction

It is important to note that one-parameter irreducible representations of Lie superalgebra are the crucial roles in constructing supersymmetrical models. The supersymmetrical algebra of BGLZ model for correlated electrons on the unrestricted 4^L-dimensional electronic Hilbert space $\bigotimes_{n=1}^{L} C^4$ is superalgebra gl(2|1) [1]. It is interesting that those models contain one symmetry-preserving free real parameter which is the Hubbard interaction parameter U. The coherent states of Lie (super)algebras are very important in the study of quantum mechanics, quantum electrodynamics, quantum optics and quantum field theory, which provide a natural link between classical and quantum phenomena and are related to the path integral formalism. One-parameter indecomposable and irreducible representations of the spl(2, 1) superalgebra have been studied [2, 3]. The purpose of the present paper is to derive further one-parameter general coherent states of the spl(2, 1) superalgebra on the basis of studying

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one-parameter irreducible representation, and discuss their properties. In the present paper we shall first construct general coherent states of the spl(2, 1) superalgebra in four irrep subspaces. Then we discuss their properties. We calculate also one-parameter matrix elements of the spl(2, 1) generators in the one-parameter general coherent-state space. In other article, we shall give a new form of the inhomogeneous differential realizations of the spl(2, 1) in one-parameter coherent-state space.

2 The spl(2, 1) One-Parameter General Coherent States and Properties

In accordance with the [4] the generators of the spl(2, 1) superalgebra read as follows:

$$\{Q_3, Q_+, Q_-, B \in \mathrm{spl}(2,1)\bar{0} \mid V_+, V_-, W_+, W_- \in \mathrm{spl}(2,1)\bar{1}\} \tag{1}$$

and satisfy the following commutation and anticommutation relations:

$$\begin{aligned}
&[Q_3, Q_\pm] = \pm Q_\pm, \qquad [Q_+, Q_-] = 2Q_3, \qquad [B, Q_\pm] = [B, Q_3] = 0,\\
&[Q_3, V_\pm] = \pm\frac{1}{2}V_\pm, \qquad [Q_3, W_\pm] = \pm\frac{1}{2}W_\pm, \qquad [B, V_\pm] = \frac{1}{2}V_\pm,\\
&[B, W_\pm] = -\frac{1}{2}W_\pm, \qquad [Q_\pm, V_\mp] = V_\pm, \qquad [Q_\pm, W_\mp] = W_\pm, \qquad [Q_\pm, V_\pm] = 0,\\
&[Q_\pm, W_\pm] = 0, \qquad \{V_\pm, V_\pm\} = \{V_\pm, V_\mp\} = \{W_\pm, W_\pm\} = \{W_\pm, W_\mp\} = 0,\\
&\{V_\pm, W_\pm\} = \pm Q_\pm, \qquad \{V_\pm, W_\mp\} = -Q_3 \pm B.
\end{aligned} \tag{2}$$

In [5] we gave a typical 4-dimensional one-parameter elementary representation of the spl(2, 1)

$$\begin{aligned}
D(V_+) &= \begin{bmatrix} 0 & 0 & 0 & 0\\ \sqrt{\alpha} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \sqrt{\alpha+1} & 0 \end{bmatrix}, \qquad
D(V_-) = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ \sqrt{\alpha} & 0 & 0 & 0\\ 0 & -\sqrt{\alpha+1} & 0 & 0 \end{bmatrix},\\
D(W_+) &= \begin{bmatrix} 0 & 0 & -\sqrt{\alpha} & 0\\ 0 & 0 & 0 & \sqrt{\alpha+1}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad
D(W_-) = \begin{bmatrix} 0 & \sqrt{\alpha} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \sqrt{\alpha+1}\\ 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned} \tag{3}$$

According to (3) we have obtained one-parameter indecomposable and irreducible representations of the spl(2, 1) superalgebra on the quotient space of $V[2,3]$

$$Y = (V/J) : \{\phi(k, \alpha_1, \alpha_2) = \phi(k, 0, \alpha_1, 0, \alpha_2, 0) \bmod J \mid k \in Z^+, \alpha_1, \alpha_2 = 0, 1\}.$$

Relabelling the basis vector $\phi(k, \alpha_1, \alpha_2)$ of the finite-dimensional irreducible representation of the spl(2, 1) superalgebra by $|N, k, \alpha_1, \alpha_2\rangle$ the actions of the generators on the basis vectors are

$$
\begin{aligned}
Q_3|N,k,\alpha_1,\alpha_2\rangle &= \left(-\frac{1}{2}N+k+\frac{1}{2}\alpha_1+\frac{1}{2}\alpha_2\right)|N,k,\alpha_1,\alpha_2\rangle,\\
B|N,k,\alpha_1,\alpha_2\rangle &= \left[\left(\frac{1}{2}+\alpha\right)N-\frac{1}{2}\alpha_1-\frac{1}{2}\alpha_2\right]|N,k,\alpha_1,\alpha_2\rangle,\\
Q_+|N,k,\alpha_1,\alpha_2\rangle &= (N-k-\alpha_1-\alpha_2)|N,k+1,\alpha_1,\alpha_2\rangle,\\
Q_-|N,k,\alpha_1,\alpha_2\rangle &= k|N,k-1,\alpha_1,\alpha_2\rangle,\\
V_+|N,k,\alpha_1,\alpha_2\rangle &= \alpha_1\sqrt{\alpha}|N,k+1,\alpha_1-1,\alpha_2\rangle\\
&\quad +(-1)^{\alpha_1}(1-\alpha_2)(N-k-\alpha_1)\sqrt{1+\alpha}|N,k,\alpha_1,\alpha_2+1\rangle,\\
V_-|N,k,\alpha_1,\alpha_2\rangle &= \alpha_1\sqrt{\alpha}|N,k,\alpha_1-1,\alpha_2\rangle\\
&\quad -(-1)^{\alpha_1}(1-\alpha_2)\sqrt{1+\alpha}k|N,k-1,\alpha_1,\alpha_2+1\rangle,\\
W_+|N,k,\alpha_1,\alpha_2\rangle &= (-1)^{\alpha_1}\alpha_2\sqrt{1+\alpha}|N,k+1,\alpha_1,\alpha_2-1\rangle\\
&\quad +(-N+k+\alpha_2)(1-\alpha_1)\sqrt{\alpha}|N,k,\alpha_1+1,\alpha_2\rangle,\\
W_-|N,k,\alpha_1,\alpha_2\rangle &= (1-\alpha_1)\sqrt{\alpha}k|N,k-1,\alpha_1+1,\alpha_2\rangle\\
&\quad +(-1)^{\alpha_1}\alpha_2\sqrt{1+\alpha}|N,k,\alpha_1,\alpha_2-1\rangle,
\end{aligned}
\tag{4}
$$

where

$$
\{|N,k,\alpha_1,\alpha_2\rangle \mid k+\alpha_1+\alpha_2\le N,\, N\in Z^+,\ k=0,1,2,\ldots,\alpha_1,\alpha_2=0,1\}
$$

and

$$
k=\begin{cases}
0,1,\ldots,N & \text{when } \alpha_1=0,\alpha_2=0,\\
0,1,\ldots,N-1 & \text{when } \alpha_1=0,\alpha_2=1,\\
0,1,\ldots,N-1 & \text{when } \alpha_1=1,\alpha_2=0,\\
0,1,\ldots,N-2 & \text{when } \alpha_1=1,\alpha_2=1.
\end{cases}
$$

The space $\{|N,k,\alpha_1,\alpha_2\rangle\}$ of the irrep N of the spl(2, 1) superalgebra is $4N$ dimensional and may be divided into four subspaces $\{|N,k,0,0\rangle\}$, $\{|N,k,0,1\rangle\}$, $\{|N,k,1,0\rangle\}$ and $\{|N,k,1,1\rangle\}$ corresponding to $(\alpha_1,\alpha_2)=(0,0)$, $(0,1)$, $(1,0)$ and (1,1), respectively. All the basis vectors $|N,k,\alpha_1,\alpha_2\rangle$ are assumed to be normalized as

$$
\binom{N-\alpha_1-\alpha_2}{k}\langle N,k,\alpha_1,\alpha_2|N,k,\alpha_1,\alpha_2\rangle=1.
$$

That is

$$
\begin{aligned}
&\binom{N}{k}\langle N,k,0,0|N,k,0,0\rangle=1, &&\binom{N-1}{k}\langle N,k,0,1,|N,k,0,1\rangle=1,\\
&\binom{N-1}{k}\langle N,k,1,0|N,k,1,0\rangle=1, &&\binom{N-2}{k}\langle N,k,1,1|N,k,1,1\rangle=1.
\end{aligned}
\tag{5}
$$

The completeness condition of the vectors of the irrep may be expressed as

$$\sum_{(\alpha_1,\alpha_2),k=0}^{N-\alpha_1-\alpha_2} \binom{N-\alpha_1-\alpha_2}{k} |N,k,\alpha_1,\alpha_2\rangle\langle N,k,\alpha_1,\alpha_2| = I.$$

That is

$$\sum_{k=0}^{N} \binom{N}{k} |N,k,0,0\rangle\langle N,k,0,0| + \sum_{k=0}^{N-1} \binom{N-1}{k} |N,k,0,1\rangle\langle N,k,0,1|$$
$$+ \sum_{k=0}^{N-1} \binom{N-1}{k} |N,k,1,0\rangle\,\langle N,k,1,0| + \sum_{k=0}^{N-2} \binom{N-2}{k} |N,k,1,1\rangle\,\langle N,k,1,1| = I, \tag{6}$$

where I is the identity operator.

From (4) we can easily derive the following formulas

$$Q_+^n\,|N,0,\alpha_1,\alpha_2\rangle = \binom{N-\alpha_1-\alpha_2}{n} n!\,|N,n,\alpha_1,\alpha_2\rangle\,.$$

That is

$$\begin{aligned} Q_+^n\,|N,0,0,0\rangle &= \binom{N}{n} n!\,|N,n,0,0\rangle\,, \\ Q_+^n\,|N,0,0,1\rangle &= \binom{N-1}{n} n!\,|N,n,0,1\rangle\,, \\ Q_+^n\,|N,0,1,0\rangle &= \binom{N-1}{n} n!\,|N,n,1,0\rangle\,, \\ Q_+^n\,|N,0,1,1\rangle &= \binom{N-2}{n} n!\,|N,n,1,1\rangle\,, \end{aligned} \tag{7}$$

where

$$\binom{N}{n} = \frac{N!}{(N-n)!n!}.$$

In terms of Bloch's method we now define one-parameter general coherent state of the spl(2, 1) by applying the exponential operator $\exp(ZQ_+)$ on the lowest-weight state $|N,0,\alpha_1,\alpha_2\rangle$ of its irrep

$$|z,\alpha_1,\alpha_2\rangle = C(z,\alpha_1,\alpha_2)\exp(zQ_+)|N,0,\alpha_1,\alpha_2\rangle, \tag{8}$$

where $C(z,\alpha_1,\alpha_2)$ is a normalization constant to be determined.

Using the formulas (7), the coherent state (8) may be rewritten as follows:

$$|z,\alpha_1,\alpha_2\rangle = C(z,\alpha_1,\alpha_2) \sum_{n=0}^{N-\alpha_1-\alpha_2} \binom{N-\alpha_1-\alpha_2}{n} z_n|N,n,\alpha_1,\alpha_2\rangle. \tag{9}$$

We require that the spl(2, 1) coherent state defined in this way are normalized in the form

$$\langle z,\alpha_1,\alpha_2|z,\alpha_1,\alpha_2\rangle = 1. \tag{10}$$

It follows from (9) and (10) that

$$C(z,\alpha_1,\alpha_2)=(1+\bar{z}z)^{-\frac{1}{2}(N-\alpha_1-\alpha_2)}. \tag{11}$$

The scalar product of two coherent states is of the form

$$\langle z',\alpha_1',\alpha_2'|z,\alpha_1,\alpha_2\rangle=C(z',\alpha_1',\alpha_2')C(z,\alpha_1,\alpha_2)(1+\bar{z}'z)^{N-\alpha_1-\alpha_2}\delta_{\alpha_1',\alpha_1}\delta_{\alpha_2',\alpha_2}. \tag{12}$$

The expansion coefficients of the coherent state $|Z,\alpha_1,\alpha_2\rangle$ may be found in terms of the complete orthonormal set $\{|N,k,\alpha_1,\alpha_2\rangle\}$.Thus, we have

$$\langle N,k,\alpha_1,\alpha_2|z,\alpha_1,\alpha_2\rangle=C(z,\alpha_1,\alpha_2)z^k. \tag{13}$$

While orthogonality is a convenient property for a set of basis vectors it is not a necessary one. The essential property of such a set is that it be complete. Since the $4N$ state vectors $\{|N,k,\alpha_1,\alpha_2\rangle\}$ of an irrep of the spl(2, 1) superalgebra are known to form a completeness orthogonal set, the set of the coherent states $\{|Z,\alpha_1,\alpha_2\rangle\}$ for the spl(2, 1) superalgebra can be shown without difficulty to form a complete set. To give a proof we need only demonstrate that the unit operator may be expressed as a suitable sum or an integral, over the complex Z plane, of projection operators of the form $|Z,\alpha_1,\alpha_2\rangle\langle Z,\alpha_1,\alpha_2|$. In order to describe such integral we introduce generally the differential element of weight area in the Z plane

$$d^2\sigma(Z,\alpha_1,\alpha_2)=\sigma(|Z|,\alpha_1,\alpha_2)d^2(Z,\alpha_1,\alpha_2)=\sigma(|Z|,\alpha_1,\alpha_2)(\mathrm{Re}\,Z)d(\mathrm{Im}\,Z). \tag{14}$$

If we set $Z=|Z|e^{i\theta}$, then we may rewrite (14) as

$$d^2\sigma(Z,\alpha_1,\alpha_2)=\sigma(|Z|,\alpha_1,\alpha_2)|Z|d|Z|d\theta. \tag{15}$$

The problem here may by changed to find the weight function $\sigma(Z,\alpha_1,\alpha_2)$ such that

$$\int d^2\sigma(z,\alpha_1,\alpha_2)|Z,\alpha_1,\alpha_2\rangle\langle Z,\alpha_1,\alpha_2|=I. \tag{16}$$

Let $|f\rangle$ and $|g\rangle$ be two arbitrary vectors, then (16) means that

$$\langle f|g\rangle=\int d^2\sigma(z,\alpha_1,\alpha_2)\langle f|Z,\alpha_1,\alpha_2\rangle\langle Z,\alpha_1,\alpha_2|g\rangle. \tag{17}$$

Substituting the definition (9) into (17) and integrating over the entire area of the complex plane we have

$$\begin{aligned}\langle f|g\rangle=2\pi\sum_{\substack{n=0\\ \alpha_1,\alpha_2}}^{N-\alpha_1-\alpha_2}&\binom{N-\alpha_1-\alpha_2}{n}\binom{N-\alpha_1-\alpha_2}{n}\\ &\times\int_0^\infty |Z|^{2n+1}\sigma(|Z|,\alpha_1,\alpha_2)(1+|Z|^2)^{-N+\alpha_1+\alpha_2}d|Z|\\ &\times\langle f|N,n,\alpha_1,\alpha_2\rangle\langle N,n,\alpha_1,\alpha_2|g\rangle.\end{aligned} \tag{18}$$

Comparing (18) with (6) we must have

$$2\pi\binom{N-\alpha_1-\alpha_2}{n}\int_0^\infty |z|^{2n+1}(1+|Z|^2)^{-N+\alpha_1+\alpha_2}\sigma(|Z|,\alpha_1,\alpha_2)d|Z|=1. \tag{19}$$

With the aid of the following integral identity,

$$\int_0^\infty \frac{x^{2n+1}}{(1+x^2)^m} dx = \frac{n!(m-n-2)!}{2(m-1)!} \tag{20}$$

and by comparing (19) with (20) we finally obtain the weight function

$$\sigma(|z|, \alpha_1, \alpha_2) = \frac{N - \alpha_1 - \alpha_2 + 1}{\pi(1+|z|^2)^2}. \tag{21}$$

We have thus shown

$$\frac{1}{\pi} \int d^2(z, \alpha_1, \alpha_2) \frac{N - \alpha_1 - \alpha_2 + 1}{(1+|z|^2)^2} |z, \alpha_1, \alpha_2\rangle\langle z, \alpha_1, \alpha_2| = 1, \tag{22}$$

which is a completeness relation for one-parameter general coherent states of the spl(2, 1) superalgebra of precisely the type desired. As a result of the above completeness relation, an arbitrary vector $|\Psi\rangle$ can be expanded in terms of the coherent states for the spl(2, 1) superalgebra. To secure the expansion of $|\Psi\rangle$ in terms of the coherent states $\{|Z, \alpha_1, \alpha_2\rangle\}$, we multiply $|\Psi\rangle$ by the expression (22) of the unit operator. We then find

$$|\Psi\rangle = \frac{1}{\pi} \int d^2(z, \alpha_1, \alpha_2) \frac{N - \alpha_1 - \alpha_2 + 1}{(1+|z|^2)^2} |z, \alpha_1, \alpha_2\rangle\langle z, \alpha_1, \alpha_2|\Psi\rangle. \tag{23}$$

3 One-Parameter Matrix Elements of the Generators

The present section will be devoted to calculating the matrix elements of the spl(2, 1) generators in the one-parameter general coherent-state space. The calculation results are as follows:

$$\begin{aligned}
&\langle z', \alpha_1', \alpha_2'|Q_3|z, \alpha_1, \alpha_2\rangle \\
&\quad = -\frac{1}{2} C(z', \alpha_1', \alpha_2') C(z, \alpha_1, \alpha_2)(N - \alpha_1 - \alpha_2) \\
&\qquad \times (1 - \bar{z}'z)(1 + \bar{z}'z)^{N-\alpha_1-\alpha_2-1} \delta_{\alpha_1',\alpha_1} \delta_{\alpha_2',\alpha_2}, \\
&\langle z', \alpha_1', \alpha_2'|Q_+|z, \alpha_1, \alpha_2\rangle \\
&\quad = C(z', \alpha_1', \alpha_2') C(z, \alpha_1, \alpha_2)(N - \alpha_1 - \alpha_2)\bar{z}'(1 + \bar{z}'z)^{N-\alpha_1-\alpha_2-1} \delta_{\alpha_1',\alpha_1} \delta_{\alpha_2',\alpha_2}, \\
&\langle z', \alpha_1', \alpha_2'|Q_-|z, \alpha_1, \alpha_2\rangle \\
&\quad = C(z', \alpha_1', \alpha_2') C(z, \alpha_1, \alpha_2)(N - \alpha_1 - \alpha_2) z(1 + \bar{z}'z)^{N-\alpha_1-\alpha_2-1} \delta_{\alpha_1',\alpha_1} \delta_{\alpha_2',\alpha_2}, \\
&\langle z', \alpha_1', \alpha_2'|B|z, \alpha_1, \alpha_2\rangle \\
&\quad = \frac{1}{2} C(z', \alpha_1', \alpha_2') C(z, \alpha_1, \alpha_2)[(1+2\alpha)N - \alpha_1 - \alpha_2](1 + \bar{z}'z)^{N-1} \delta_{\alpha_1',\alpha_1} \delta_{\alpha_2',\alpha_2}, \\
&\langle z', \alpha_1', \alpha_2'|V_+|z, \alpha_1, \alpha_2\rangle \\
&\quad = (-1)^{\alpha_1}(N - \alpha_1)\sqrt{1+\alpha} C(z', \alpha_1', \alpha_2') C(z, \alpha_1, \alpha_2) \\
&\qquad \times (1 + \bar{z}'z)^{N-\alpha_1-1} \delta_{\alpha_1',\alpha_1} \delta_{\alpha_2',\alpha_2+1}
\end{aligned} \tag{24}$$

$$+ C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)\sqrt{\alpha}\bar{z}'(1 + \bar{z}'z)^{N-\alpha_2-1}\delta_{\alpha_1',\alpha_1-1}\delta_{\alpha_2',\alpha_2},$$

$$\langle z', \alpha_1', \alpha_2'|V_-|z, \alpha_1, \alpha_2\rangle$$
$$= C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)\sqrt{\alpha}(1 + \bar{z}'z)^{N-\alpha_2-1}\delta_{\alpha_1',\alpha_1-1}\delta_{\alpha_2',\alpha_2}$$
$$- (-1)^{\alpha_1}(N - \alpha_1)\sqrt{1+\alpha}C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)z(1 + \bar{z}'z)^{N-\alpha_1-1}\delta_{\alpha_1',\alpha_1}\delta_{\alpha_2',\alpha_2+1},$$

$$\langle z', \alpha_1', \alpha_2'|W_+|z, \alpha_1, \alpha_2\rangle$$
$$= (N - \alpha_2)\sqrt{\alpha}C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)(1 + \bar{z}'z)^{N-\alpha_2-1}\delta_{\alpha_1'+1,\alpha_1}\delta_{\alpha_2',\alpha_2}$$
$$+ (-1)^{\alpha_1}C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)\sqrt{1+\alpha}\bar{z}'(1 + \bar{z}'z)^{N-\alpha_1-1}\delta_{\alpha_1',\alpha_1}\delta_{\alpha_2',\alpha_2-1},$$

$$\langle z', \alpha_1', \alpha_2'|W_-|z, \alpha_1, \alpha_2\rangle$$
$$= (-1)^{\alpha_1}C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)\sqrt{1+\alpha}(1 + \bar{z}'z)^{N-\alpha_1-1}\delta_{\alpha_1',\alpha_1}\delta_{\alpha_2',\alpha_2-1}$$
$$- (N - \alpha_2)\sqrt{\alpha}C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)z(1 + \bar{z}'z)^{N-\alpha_2-1}\delta_{\alpha_1',\alpha_1+1}\delta_{\alpha_2',\alpha_2}.$$

In evaluating the matrix elements, one needs only use (3), (5) and (9), for example,

$$\begin{aligned}
&\langle z', \alpha_1', \alpha_2'|Q_3|z, \alpha_1, \alpha_2\rangle \\
&\quad = C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2) \\
&\qquad \times \sum_{m=0}^{N-\alpha_1'-\alpha_2'}\sum_{n=0}^{N-\alpha_1-\alpha_2}\binom{N-\alpha_1'-\alpha_2'}{m}\binom{N-\alpha_1-\alpha_2}{n} \\
&\qquad \times (\bar{z}')^m z^n\langle N, m, \alpha_1', \alpha_2'|Q_3|N, n, \alpha_1, \alpha_2\rangle \\
&\quad = C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)\sum_{n=0}^{N-\alpha_1-\alpha_2}\binom{N-\alpha_1-\alpha_2}{n}\left\{-\frac{1}{2}(N-\alpha_1-\alpha_2)+n\right\}(\bar{z}'z)^n \\
&\quad = C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2)\left\{\frac{1}{2}(N-\alpha_1-\alpha_2)\sum_{n=0}^{N-\alpha_1-\alpha_2}\binom{N-\alpha_1-\alpha_2}{n}(\bar{z}'z)^n\right. \\
&\qquad \left. - (N-\alpha_1-\alpha_2)\sum_{n=0}^{N-\alpha_1-\alpha_2-1}\binom{N-\alpha_1-\alpha_2-1}{n}(\bar{z}'z)^n\right\} \\
&\quad = -\frac{1}{2}(N-\alpha_1-\alpha_2)C(z', \alpha_1', \alpha_2')C(z, \alpha_1, \alpha_2) \\
&\qquad \times (1-\bar{z}'z)(1+\bar{z}'z)^{N-\alpha_1-\alpha_2-1}\delta_{\alpha_1',\alpha_1}\delta_{\alpha_2',\alpha_2}.
\end{aligned} \tag{25}$$

4 Conclusion

We have constructed one-parameter general coherent states of the spl(2, 1) superalgebra. We have discussed the orthogonality and completeness relations for the coherent state of the spl(2, 1) superalgebra. We have calculated one-parameter matrix elements of the spl(2, 1) generators in the one-parameter general coherent-state space. On the basis, we can study new one-parameter inhomogeneous differential realizations of the spl(2, 1) superalgebra.

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